Mathematician: _____

Summer Work -

Algebra to Geometry Summer 2025

This is optional. It will not be collected or assessed in any form next year, but it would be helpful to keep your Algebra skills sharp.

Questions?

Email: mstirling@hollandhall.org

Combining Like Terms

Example: Simplify $8x^2 + 16xy - 3x^2 + 3xy - 3x$

$8x^2 - 3x^2 + 16xy + 3xy - 3x$ *Identify &/or Group Like Terms*

$5x^2 - 3x + 19xy$

Simplify

$6x + 11y - 4x + y$	$-3p - 4t - 5t - 2p$	$3x^2y - 5xy^y + 6x^2y$
$-5m + 3q + 4m - q$	$9x - 22y + 18x - 3y$	$5x^2 + 2xy - 7x^2 + xy$

Solving Equations with variables on both sides

Example: Solve $6a - 12 = 5a + 9$

$a - 12 = 9$ *subtract 5a from both sides*

Solve each equation $a = 21$ *add 12 to both sides*

$3x + 5 = 2x + 11$	$8m + 1 = 7m - 9$	$11q - 6 = 3q + 8q$
$-14 + 3a = 10 - a$	$-2t + 10 = -t$	$-7x + 7 = 2x - 11$

Literal Equations

Solve $2p = kx - q$ for x

$2p = kx - q$ *Solve for x by isolating the variable x*

$2p + q = kx$ *Add q to each side*

$\frac{2p + q}{k} = x$ *Divide each side by k*

$x = \frac{2p + q}{k}$

Solve for the indicated variable.

$ax - c = b$; solve for x	$2x + 4y = 8$; solve for y	$\frac{2}{3}y + 4x = -2$; solve for y

Solving Inequalities & Graphing

Example: Solve. & Graph

$5x - 4 \geq 4x + 6$

$x - 4 \geq 6$

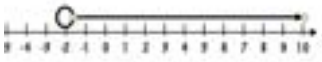
$x \geq 10$

Closed circle at 10; arrow going to the right (greater)

$10 - 7x < 24$

$-7x < 14$

$x > -2$



Open circle at -2; arrow going to the right (greater)

When you multiply or divide each side of an inequality by a negative number, you must reverse the inequality symbol to maintain a true statement.

Solve & Graph.

$-x + 2 > 7$	$-5 + m \leq 4$	$z + 6 > -2$
$-16 - 8x \geq 0$	$x - 5 < 4$	$-3x + 4 \leq -5$
$8x - 6 \geq 10$	$9(2x - 5) - 3 < 7x - 4$	$9x - 11 > 6x - 9$

Calculating Slope

Example: Find the slope of a line passing through (3, - 9) and (2, -1).

$m = \frac{y_2 - y_1}{x_2 - x_1}$ *Formula for slope*

$m = \frac{-1-(-9)}{2-3} = \frac{-1+9}{-1}$ *Substitute values and simplify*

Find slope. $m = \frac{8}{-1} = -8$ *Slope is -8*

(4 , 1) (3, 6)	(5, 6) (9, 8)	(-1, 7) (-3, 18)
(-8, 0) (5, -2)	(0, - 4) (7, 3)	(-6, -4) (1, 10)

Finding the equation of a line (given a point and y-intercept)

Example Find an equation of the line that passes through the point (3,4) and has a y-intercept of 5

y = mx + b	*Slope-intercept form*
4 = 3m + 5	*Substitute 5 for b, 3 for x, and 4 for y.*
-1 = 3m	*Subtract 5 from both sides*
-⅓ = m	*Divide each side by 5*

Find the equation of the line. y = -⅓ x + 5 *With slope -⅓ & y-intercept of 5, this is the line's equation*

(2,1); b = 5	(7, 0); b = 13	(-5, 3); b = -12
(-3, -3); b = -2	(-3, 10); b = 8	(-1, 4); b = -8

Finding the equation of a line (given a point and the slope)

Example Find an equation of the line that passes through the point (1,2) and has a slope of -3

$y - y_1 = m(x - x_1)$	*Point-Slope Form*
$y - 2 = -3(x - 1)$	*Substitute 2 for y_1 1 for x_1 and -3 for m.*
$y - 2 = -3x + 3$	*Distribute the -2*
$y = -3x + 5$	*Add 2 to both sides*

Find the equation of the line in slope-intercept form.

(2,3); m = -4	(-1, 5); m = 2	(4, 6); m = -½
(-3, - 4); m = ⅔	(0, 4); m = -3/2	(5, 0); m = -4

Finding the equation of a line (given two points)

Example Write an equation of the line that passes through the points (4,8) and (3,1).

$m = \frac{1-8}{3-4}$	*Substitute values into the formula for slope*
$m = \frac{-7}{-1} = 7$	*Simplify*
y- 1 = 7(x - 3)	*Select either point and substitute values of point and slope into point-slope form*
y- 1 = 7x -21	*Distribute the 7*
y = 7x - 20	*Add 1 to each side to get the equation of a line in slope-intercept form*

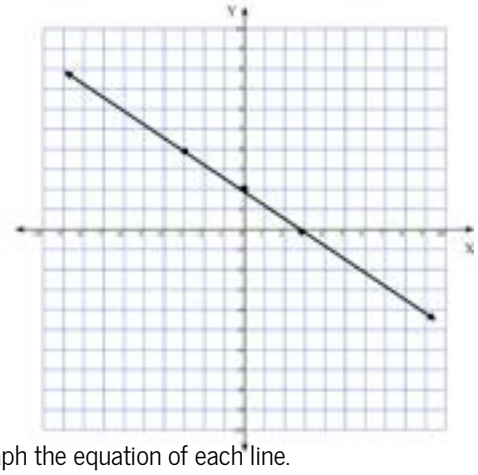
Find the equation of the line in slope-intercept form.

(6, -3) (1, 2)	(5, -1) (4, -5)	(-3, -7) (0, 8)
(-7, 9) (-5, 3)	(-2, 4) (3, -6)	(1, 2) (-1, -4)

Standard Form of a Line (Ax + By = C)

Example Graph a line in standard form 2x+3y = 6

Option 1:	Change to slope-intercept form	Option 2	Keep in Standard form
2x + 3y = 6 3y = -2x + 6 y = -⅔ x + 2 m = -⅔ and b = 2	*Subtract 2x from each side* *Divide everything by 3* *Graph a y-intercept at 2 & a slope of -⅔*	2x + 3y = 6 m = $\frac{-A}{B} = \frac{-2}{3}$ b = $\frac{C}{B} = \frac{6}{3} = 2$	Slope in standard form $\frac{-A}{B}$ y-intercept in stand form b= $\frac{C}{B}$ Graph using a slope of -⅔ & y-intercept of 2



Graph the y-intercept of 2, then go down 2 and right 3 to find another point on the line.
You can also go up 2 and left 3; connect the points to make a line.

Graph the equation of each line.

4x + 5y = 10		x -4y = 8	
2x - 3y = 5		3x - 4y = -12	

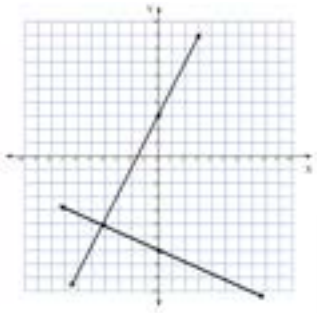
Solving Systems of Equations (by graphing or substitution)

Example Solve the system y = 2x + 5 and y = -½ x -4

By Graphing

Graph y = 2x + 3
y-intercept of 3
slope of 2

Graph y = -½ x - 7
y-intercept of -7
slope of -½



(-4, -5) Coordinates for solution

By substitution

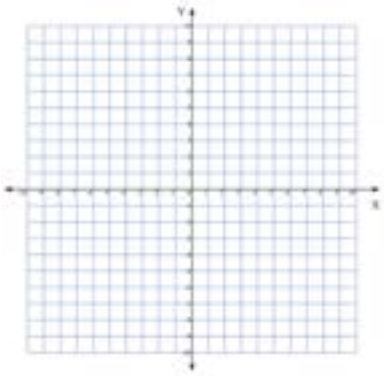
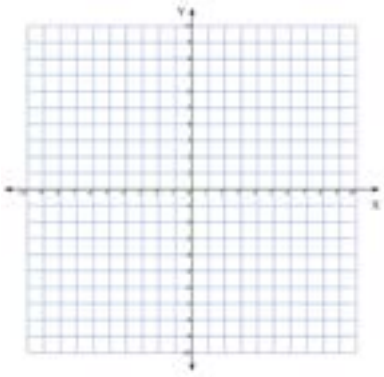
Given y= 2x + 3 & y= -½ x -7

2x + 3 = -½ x - 7 *Substitute in place of y*
2 ½x + 3 = -7 *Add ½ x to each side*
2 ½ x = -10 *Subtract 3 from each side*
x = -4 *Divide each side by 2 ½*

y= 2(-4) + 3 *Substitute (-4) in place of x*
y = -8 + 3 *Simplify*
y = -5 *Combine like terms to find y*

(-4, -5) *Coordinates for solution*

Solve each system by graphing or substitution

y= 2x + 4 -3x + y = -9		y = x - 1 x + y = 3
4x + y = 0 x + 2y = -7		½ x + 2y = 12 X - 2y = 6

Solve Systems of Equations (by elimination)

Example

$x - y = 4$
$2x + 2y = -4$ *Given*

$2(x - y = 4)$
$2x + 2y = -4$ *Multiply by 2 to eliminate the y*

$2x - 2y = 8$
$\underline{2x + 2y = -8}$
$4x = 0$ *Add the 2 equation*
$x = 0$ *Divide each side by 4*

$2(0) + 2y = -4$ *Substitute 0 for x in*
$2y = -4$ *either equation; simplify*
$y = -2$ *Divide each side by 2*

$(0, -2)$ *Solution to system*

Solve each system by elimination.

$x - y = 1$ $x + y = 3$	$3x + 4y = 19$ $3x + 6y = 33$	$2x - 3y = 9$ $-5x - 3y = 30$
$5x - y = -6$ $-x + y = 2$	$6x - 3y = 6$ $6x + 8y = -16$	The sum of two numbers is 28. Their difference is 4. What are the two numbers?

Solving Proportions

Example $\frac{x}{8} = \frac{3}{4}$

$4x = 8 \cdot 3$ *Cross multiply*

$4x = 24$

$x = 6$

$\frac{6}{x+4} = \frac{2}{9}$

$6 \cdot 9 = 2(x + 4)$ *Cross Multiply*

$54 = 2x + 8$ *Simplify both sides*

$46 = 2x$ *Subtract 8 from both sides*

$x = 23$ *Divide each side by 2*

Solve each proportion to find the value of the given variable.

$\frac{y}{40} = \frac{3}{8}$	$\frac{3}{p-6} = \frac{1}{p}$	$\frac{3}{8} = \frac{3}{2d}$
$\frac{r}{3r+1} = \frac{2}{3}$	$\frac{3}{m+4} = \frac{9}{14}$	$\frac{w}{4} = \frac{9}{w}$

Property of Exponents

Property		**Example**
Product of Powers	$a^m \cdot a^n = a^{m+n}$	$x^4 \cdot x^2 = x^6$
Power of a Power	$(a^m)^n = a^{m \cdot n}$	$(x^4)^2 = x^8$
Power of a Product	$(ab)^m = a^m b^m$	$(2x)^3 = 8x^3$
Negative Power	$a^{-n} = \frac{1}{a^n}$	$x^{-3} = \frac{1}{x^3}$
Zero Power	$a^0 = 1$	$4^0 = 1$
Quotient of Powers	$\frac{a^m}{a^n} = a^{m-n}$	$\frac{x^{10}}{x^4} = x^6$
Power of Quotients	$(\frac{a}{b})^m = \frac{a^m}{b^m}$	$(\frac{x}{y})^3 = \frac{a^3}{b^3}$

Simplify each exponent. Answers should be written using positive exponents.

$g^5 \cdot g^{11}$	$(b^6)^3$	w^{-7}
$\frac{y^{12}}{y^8}$	$(3x^7)(-5x^3)$	$(-4a^5b^0c)^2$
$-6(x^2y^3)^4$	$(-18mn^4)(-\frac{1}{6}mn^2)$	$\frac{16x^5y^2}{2x^3y^3}$

Polynomial Operations

Add or Subtract like terms	Distributing	Multiplying binomials &/or trinomials
$(7x^2 + 4x - 3) - (-5x^2 - 3x + 2)$	$-2x(5x + 11)$	$(7x - 3)(3x + 7)$ *Multiply everything in the 1st binomial times the 2nd*
$7x^2 - (-5x^2) + 4x - (-3x) - 3 - (2)$	$-10x^2 - 22x$	$7x \cdot 3x + 7x \cdot 7 - 3 \cdot 3x - 3 \cdot 7$ ***F**irst-**O**utside-**I**nside-**L**ast (FOIL)*
$12x^2 + 7x - 5$		$21x^2 + 49x - 9x - 21$ *Simplify*
		$21x^2 + 40x - 21$ *Combine like terms*

Simplify each polynomial

$(2x + 3y) + (4x + 9y)$	$(7x^2 + x + 1) - (3x^2 - 4x - 3)$	$(7a^2 - a + 4) - (3a^2 - 4a - 3)$
$-3x(8x^2 - 3x + 1)$	$-10pq(3pq + 4p - 5q^2)$	$5w(w^2 - 7w + 3) - 2w(2w^2 - 5w + 2)$
$(x + 4)(x - 7)$	$(5x - 2y)(3x + 9y)$	$(z + 5)(4z - 6)$

Factoring Polynomials ($ax^2 + bx + c$)

Examples:

Factoring out GCF	Difference of squares	Perfect Square Trinomials	Trinomials
$6x^2 + 21x$	$x^2 - 64$	$4x^2 + 12x + 9$	$3x^2 + 7x + 2$
$3x(2x + 7)$	$(x + 8)(x - 8)$	$(2x + 3)^2$	$(3x + 1)(x + 2)$

Factor completely.

$6e^3f - 11ef$	$y^2 - 5y - 84$	$6x^2 + 7x + 2$
$6z^2 - 5z - 4$	$75x^2 - 147y^2$	$x^2 - 25$
$x^2 - 6x + 9$	$16c^2 + 72cd + 81d^2$	$x^4 - 16$

Solving Quadratics

Example	$x^2 + 3x^2 = 10$	
	$x^2 + 3x^2 - 10 = 0$	*Subtract 10 from both sides so the quadratic is equal to 0*
	$(x - 5)(x+ 2) = 0$	*Factor (see previous section on different kinds of factoring)*
	$x - 5 = 0$ and $x + 2 = 0$	*Use Zero Product Property and put each binomial equal to 0*
	$x = 5$ and $x = -2$	*Solve both for x.*

Solve each quadratic to find the possible values of x.

$3x^2 - 12 = 0$	$6x^2 - 5x + 1= 0$	$x^2 + 7x = 18$
$x^2 + 11x = 80$	$2x^2 = x + 15$	$3x^3 + 3x^2 - 60x = 0$

Simplifying Radicals

An expression is in simplest radical form when:

1.) there is no integer under the radical sign with a perfect square

2.) there are no fractions under the radical sign

3.) there are no radical in the denominator

Examples:

$\sqrt{20}$

$\sqrt{4}\cdot\sqrt{5}$

$2\sqrt{5}$

$\sqrt{\frac{13}{49}}$

$\frac{\sqrt{13}}{\sqrt{49}}$

$\frac{\sqrt{13}}{7}$

$\sqrt{\frac{9}{24}}$

$\frac{\sqrt{9}}{\sqrt{24}}$

$\frac{3}{\sqrt{4}\cdot\sqrt{6}} = \frac{3}{2\cdot\sqrt{6}}$

$\frac{3}{2\cdot\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}}$

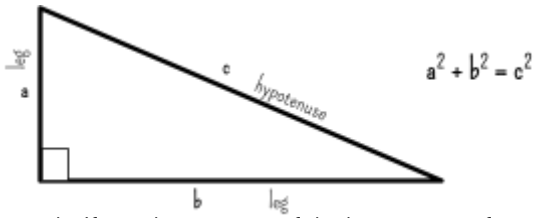
$\frac{3\sqrt{6}}{2\sqrt{36}} = \frac{3\sqrt{6}}{2\cdot 6}$

$\frac{3\sqrt{6}}{12} = \frac{\sqrt{6}}{4}$

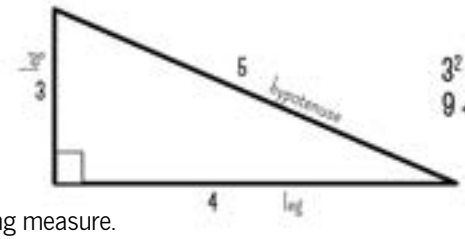
Express the following in simplest radical form.

$\sqrt{121}$	$\sqrt{40}$	$\sqrt{72}$
$\sqrt{\frac{25}{36}}$	$\sqrt{\frac{27}{45}}$	$\sqrt{\frac{50}{75}}$
$\frac{\sqrt{32}}{\sqrt{5}}$	$\sqrt{320}$	$\sqrt{5} \cdot \sqrt{60}$

Pythagorean Theorem



$a^2 + b^2 = c^2$



$3^2 + 4^2 = 5^2$
$9 + 16 = 25$
$25 = 25$

Example: If c is the measure of the hypotenuse of a right triangle, find each missing measure.

a = 6, c = 10, b = ____

$a^2 + b^2 = c^2$

$6^2 + b^2 = 10^2$

$36 + b^2 = 100$

$b^2 = 64$

$b = 8$

If c is the measure of the hypotenuse of a right triangle, find each missing measure.

a= 5, b = 12, c = _____	a = 20, b = _____ & c = 29
a = $\sqrt{6}$, b = $\sqrt{19}$, c = _____	a = _____, b = 12, & c = 15
a = 8, b = 15, c = _____	a = _____, b = 40, & c = 41

Function of the day

$f(x) = \frac{2}{3}x - 1$

Type of function:

Quadrants:

Domain:

Range:

x -intercept:

y-intercept

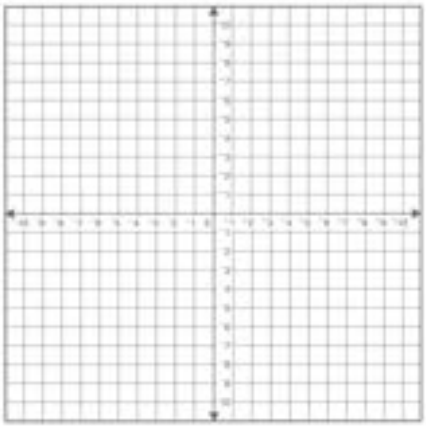
slope:

increasing or decreasing:

continuous:

parent function:

proportional:



$f(x) = -3x + 7$

Type of function:

Quadrants:

Domain:

Range:

x -intercept:

y-intercept

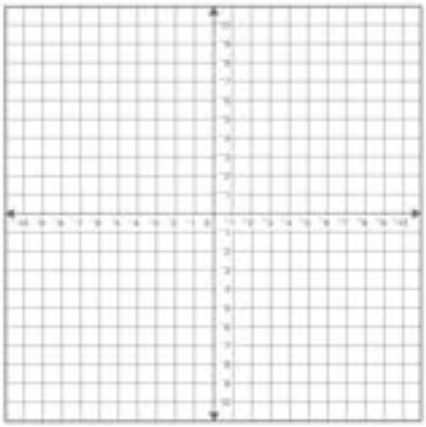
slope:

increasing or decreasing:

continuous:

parent function:

proportional:



$f(x) = x^2 - 2$

type of function:

quadrants:

domain:

range:

x -intercept:

y-intercept:

axis of symmetry:

vertex:

maximum or minimum:

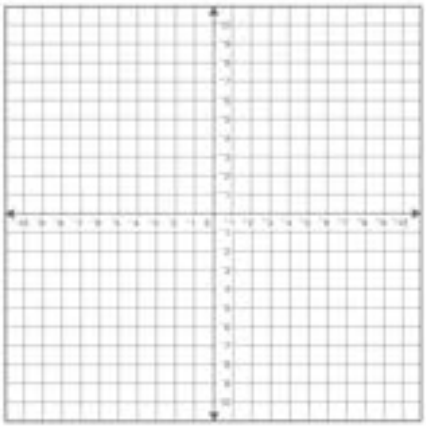
opens upward or downward:

increasing interval:

decreasing interval:

parent function:

translated:



$f(x) = -2x^2 + 1$

type of function:

quadrants:

domain:

range:

x -intercept:

y-intercept:

axis of symmetry:

vertex:

maximum or minimum:

opens upward or downward:

increasing interval:

decreasing interval:

parent function:

translated: